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Effects of non-uniform wall temperature or mass transfer in finite sections of an inclined plate on the MHD natural convection flow in a temperature stratified high-porosity medium

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Abstract

MHD Natural convection from a non-isothermal inclined surface with multiple suction/injection slots embedded in a thermally stratified high-porosity medium has been studied. The non-linear coupled parabolic partial differential equations have been solved numerically by using an implicit finite-difference scheme. The non-uniform wall temperature or the surface mass transfer in finite sections of the surface strongly affect the heat transfer and the skin friction. When the surface in the slots is cooled, the direction of the heat transfer changes. Both the skin friction and the heat transfer increase due to suction but decrease due to injection. The heat transfer, in general, changes significantly with the stratification and magnetic parameters, in the slot near the trailing edge of the plate.

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1. Introduction

Natural convection phenomena arise in nature and in industries when a heated surface or substance is brought into contact with a mass of fluid. The temperature changes cause density variations leading to buoyancy forces. This process of heat transfer is encountered in atmospheric and oceanic circulations, in the handling of spent nuclear reactor fuel assemblies, in the design of solar energy collectors, in the process of frost formation involving low-temperature surfaces, etc. Natural convection from a vertical surface in a constant-density medium has been widely studied. Gebhart et al. [1] have presented an overview of the natural convection flows. In many free convection flows which occur in nature and industry, the density of the medium is often stably stratified with lighter fluid overlying denser fluid. The fluid is stably stratified with temperature increasing with height except in the case of water between zero and 4 °C. Thermal stratification is important in lakes, rivers and the

sea and in condensers of power plants and various industrial units. The natural convection flow over a heated vertical surface with uniform temperature immersed in an ambient fluid whose temperature increases linearly with height was first studied by Eichhorn [2] who solved the governing partial differential equations by using a series solution method, wherein only three terms in the series expansion were used. Fujii et al. [3] considered the non-linear thermal stratification and found that four terms are required in the series expansion method. Yang et al. [4] showed that the similarity solution exists for the physically unrealistic situation where the temperature of the fluid decreases with height. However, a similarity solution does exist when the wall and the ambient temperature increase with height. Jaluria and Gebhart [5] have obtained similarity solution for the constant heat flux case when the temperature of both the surface and the ambient fluid increase with height. Further, Chen and Eichhorn [6,7] and, Venkatachala and Nath [8] have investigated the same problem by using a local non-similarity method and an implicit finite-difference scheme, respectively. On the other hand, Semenov [9], Kulkarni et al. [10] and, Henkes and Hoogendoorn [11] have obtained similarity solutions. The similarity solutions have one major

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Nomenclature

A	dimensionless constant	u, v	velocity components $\text{m}\cdot\text{s}^{-1}$
B	magnetic field	U^*	hypothetical velocity $\text{m}\cdot\text{s}^{-1}$
C^*	inertia coefficient	v	characteristic velocity $\text{m}\cdot\text{s}^{-1}$
C_f	skin friction co-efficient	<i>Greek letters</i>	
C_p	specific heat at constant pressure . . . $\text{kJ}\cdot\text{kg}^{-1}\cdot\text{K}$	α_e	effective thermal diffusivity $\text{m}^2\cdot\text{s}^{-1}$
E	electric field	η, ξ	dimensionless coordinates
Ec	Eckert number	μ	dynamic viscosity $\text{kg}\cdot\text{m}^{-1}\cdot\text{s}^{-1}$
F, G	dimensionless axial velocities $\text{m}\cdot\text{s}^{-1}$	μ_0	magnetic permeability
Gr	Grashof number	ν	kinematic viscosity $\text{m}^2\cdot\text{s}^{-1}$
K^*	permeability of the porous medium	ρ	density $\text{kg}\cdot\text{m}^{-3}$
K_e	effective thermal conductivity	ψ	Darcy number
k	thermal conductivity $\text{W}\cdot\text{m}^{-1}\cdot\text{K}$	σ	electrical conductivity
L	characteristic length m	Ω	angle of inclination
M	magnetic parameter	Γ	dimensional inertia coefficient
Nu	Nusselt number	<i>Subscripts</i>	
p	pressure $\text{Pa}, \text{N}\cdot\text{m}^{-2}$	i	initial conditions
Pr	Prandtl number	w	condition at the wall
Re_x	Reynolds number, based on x	∞	conditions at infinity
Re_m	magnetic Reynolds number		
S	thermal stratification parameter		
T	temperature K		

drawback. These are valid only if the surface temperature is higher than the ambient temperature at all heights of the wall. Angirasa and Srinivasan [12] have investigated the natural convection over a vertical surface embedded in a thermally stratified medium due to the combined effects of the buoyancy forces caused by the heat and mass diffusion and obtained the solution of the boundary layer equations by using an explicit finite-difference method. It was shown that the boundary layer approximations break down in the case of opposing buoyant mechanism and high levels of ambient thermal stratification. To overcome these difficulties, Angirasa and Srinivasan [13] have subsequently solved the Navier–Stokes equations along with the energy equation by employing the alternating direction implicit (ADI) method.

Natural convection flow over a vertical surface embedded in porous media also occurs in several engineering problems such as those encountered in the design of pebble-bed nuclear reactors, catalytic reactors, and compact heat exchangers, in geothermal energy conversion, in the use of fibrous materials, in the thermal insulation of buildings, in the heat transfer from storage of agricultural products which generate heat as a result of metabolism, in petroleum reservoirs, in nuclear wastes, etc. Excellent reviews of the natural convection flows in porous media have been presented by Combarous and Bories [14], Catton [15], Bejan [16,17], and Tien and Vafai [18]. The natural convection flow over a vertical heated surface in a porous medium has been studied by Bejan and Khair [19], Nakayama and Koyama [20], and Kaviany and Mittal [21]. The non-Darcy effects on the natural convection boundary layer flow on an isothermal vertical flat plate em-

bedded in a high-porosity medium was considered by Chen et al. [22]. The effect of the ambient thermal stratification on the problem studied by Chen et al. [22] was investigated by Singh and Tiwari [23], and Chen and Lin [24]. Chamkha [25] has extended the analysis of Chen and Lin [24] to include the effects of the magnetic field. Some more recent studies on this topic were carried out by Minto et al. [26], Yin [27], and Rees and Pop [28].

The mass transfer in a finite section of the surface is of interest in many engineering problems. The slot suction/injection strongly influences the boundary layer development on the surface of the body and can delay separation. Also, the heat transfer and surface skin friction can be controlled by wall cooling/heating of the wall in a slot. Uniform wall temperature or mass transfer in a slot introduces finite discontinuity at the leading and the trailing edge of the slot which causes considerable difficulties in the numerical computation. These discontinuities can be avoided by choosing a non-uniform mass transfer or wall temperature distribution in the slot.

The aim of this analysis is to investigate the effects of heating/cooling of certain sections of the surface or of suction/injection slots, on the natural convection flow over an inclined surface embedded in a stably thermally stratified high-porosity medium. The non-Darcian effects as well as the viscous and Ohmic dissipation terms are included in the analysis. The magnetic field is applied normal to the surface of the plate which is electrically insulated. The non-linear coupled parabolic partial differential equations governing the flow have been solved numerically by using an implicit finite-difference scheme similar to that of Blottner [29]. The

results have been compared with those of Eichhorn [2], Chen and Eichhorn [6], Venkatachala and Nath [8], Chen and Lin [24] and Chamkha [25].

2. Problem formulation

We consider the steady laminar natural convection flow of an incompressible viscous electrically conducting fluid on a heated inclined finite plate maintained at a temperature $T_w(x)$ in the interval $[x_i, x_j]$ and at a temperature T_1 in the remaining portion of the plate. The surface mass transfer is applied only in the interval $[x_i, x_j]$. The plate is placed in a stably thermally stratified high-porosity medium. The temperature of the ambient medium is $T_\infty(x)$, which is assumed to increase linearly with height. Fig. 1 shows the physical model and the coordinate system. Using the Bousinesq approximation, all fluid properties are assumed to be constant except for the density changes which give rise to the buoyancy forces in the momentum equation. The viscous and Ohmic dissipation terms have been included in the analysis. The magnetic field \mathbf{B} is applied in the y -direction,

normal to the surface which is electrically non-conducting. It is assumed that the magnetic Reynolds number $Re_m = \mu_0 \sigma v L \ll 1$, where μ_0 and σ are, respectively, the magnetic permeability and electrical conductivity, and v and L are the characteristic velocity and length, respectively. Under these conditions, it is possible to neglect the induced magnetic field as compared to the applied magnetic field. Since there is no applied or polarized voltage imposed on the flow field, the electric field $\mathbf{E} = 0$. Hence only the applied magnetic field contributes to the Lorentz force, which acts in x -direction along the plate. Since the plate is inclined, both the streamwise pressure gradient term and the buoyancy force terms exist, but they have different magnitudes depending on the inclination angle $(\Omega/2)$, of the surface from the vertical. The buoyancy streamwise pressure gradient term can be neglected in comparison to the buoyancy force term if the condition $\tan(\Omega/2) \ll Re_x^{1/2}/\eta_\infty$ is satisfied [30]; where η_∞ is the edge of the boundary layer, $Re_x = U^*x/v$ is the local Reynolds number, U^* is the hypothetical velocity and v is the kinematic viscosity. For laminar boundary layer flows, Re_x lies between 10^3 to 10^5 and $\eta_\infty = 10$. For $Re_x = 10^3$, $\Omega/2 < 45^\circ$ and for $Re_x = 10^5$, $\Omega/2 < 80^\circ$. Under the above assumptions, the equations of continuity, momentum and energy under the boundary layer approximations governing the natural convection flow on the inclined plate can be expressed as:

$$u_x + v_y = 0 \tag{1}$$

$$\begin{aligned} \varepsilon^{-2}(uu_x + vv_y) \\ = \varepsilon^{-1}vu_{yy} + g\beta(T - T_\infty(x))\cos(\Omega/2) - \sigma \mathbf{B}^2 u/\rho \\ - v(u/K^*) - C^*(u^2/\rho) \end{aligned} \tag{2}$$

$$uT_x + vT_y = \alpha_e T_{yy} + (v/C_p)u_y^2 + \sigma \mathbf{B}^2 u^2/(\rho C_p) \tag{3}$$

The boundary conditions are the no-slip conditions at the surface and ambient conditions far away from the surface and these can be expressed as;

$$\begin{aligned} u(x, 0) = 0, \quad v(x, 0) = v_w(x), \quad T(x, 0) = T_w(x) \\ \text{for } x_i \leq x \leq x_j \\ v(x, 0) = 0, \quad T(x, 0) = T_1 \quad \text{for } x < x_i, \quad x > x_j \\ u(x, \infty) = 0, \quad T(x, \infty) = T_\infty(x) = T_0 + ax \cos(\Omega/2) \\ T_1 > T_0, \quad a > 0 \\ u(0, y) = 0, \quad T(0, y) = T_0 \quad y > 0 \end{aligned} \tag{4}$$

Here x and y are the distances along and perpendicular to the surface, respectively; u and v are the velocity components along the x and y directions, respectively; T is the temperature; ε is the porosity of the medium; g is the acceleration due to gravity; β is the volumetric coefficient of thermal expansion; $T_\infty(x) = T_0 + ax$, $a > 0$, is the ambient fluid temperature and T_0 is the ambient fluid temperature at $x = 0$; a is the slope of the ambient temperature and $a > 0$ for a stably stratified fluid; K^* is the permeability of

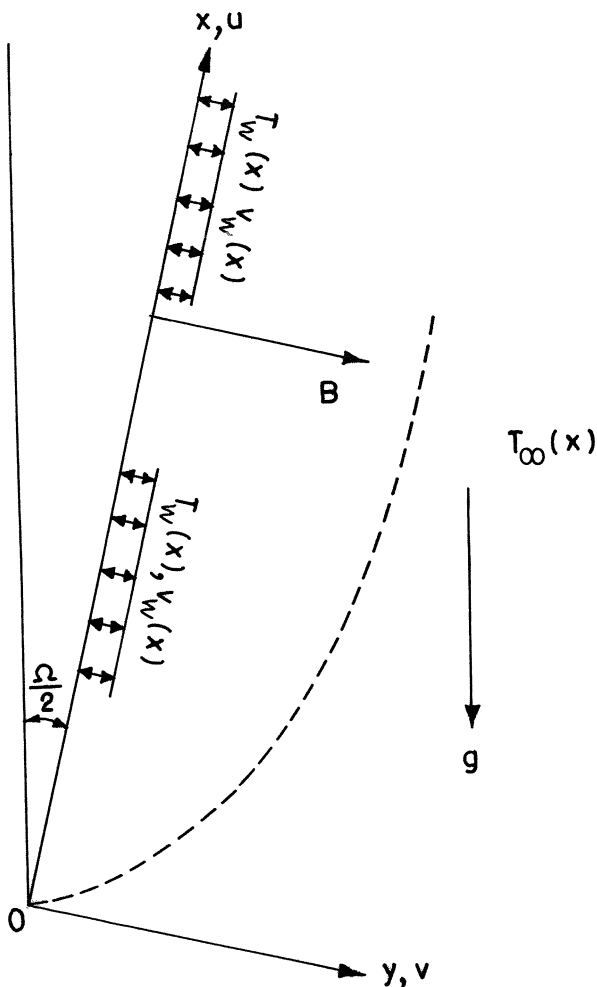


Fig. 1. Physical model and coordinate system.

the medium; C^* is the inertia coefficient; $\alpha_e (= K_e/\rho C_p)$ is the effective thermal diffusivity of the porous medium; ρ is the density of the fluid; C_p is the specific heat of the fluid at a constant pressure; K_e is the effective thermal conductivity of the fluid; T_1 ($T_1 > T_0$) is a constant wall temperature except in the slot; $\Omega/2$ is the inclination of the plate from the vertical; \mathbf{B} is the magnetic field; the subscripts x and y denote partial derivatives with respect to x and y , respectively; and the subscripts w and ∞ denote conditions at the wall and in the ambient fluid, respectively.

It is convenient to reduce Eqs. (1)–(3) in a dimensionless form by using the following transformations;

$$\begin{aligned}\xi &= x/L, & \eta &= Gr^{1/4}y/L \\ F &= Gr^{-1/2}uL/v, & G &= Gr^{-1/4}vL/v \\ \theta &= (T - T_\infty(x))/(T_1 - T_0) \\ Gr &= g\beta(T_1 - T_0)L^3/v^2 \\ Pr &= \nu/\alpha_e \\ M^2 &= \sigma \mathbf{B}^2 L^2 / (\mu Gr^{1/2}), & \psi &= K^* Gr^{1/2} / L^2 \\ \Gamma &= C^* L \\ Ec &= \nu^2 Gr / [C_p L^2 (T_1 - T_0)] \\ S &= aL \cos(\Omega/2) / (T_1 - T_0)\end{aligned}\quad (5)$$

Consequently, Eqs. (1)–(3) reduce to;

$$\begin{aligned}\partial F / \partial \xi + \partial G / \partial \eta &= 0 \\ \varepsilon^{-2} (F \partial F / \partial \xi + G \partial F / \partial \eta) \\ &= \varepsilon^{-1} \partial^2 F / \partial \eta^2 + \theta \cos(\Omega/2) - (\psi^{-1} + M^2) F - \Gamma F^2\end{aligned}\quad (6)$$

$$\begin{aligned}F \partial \theta / \partial \xi + G \partial \theta / \partial \eta \\ = Pr^{-1} \partial^2 \theta / \partial \eta^2 - SF + Ec [(\partial F / \partial \eta)^2 + M^2 F^2]\end{aligned}\quad (8)$$

The boundary conditions (4) can be re-written as;

$$\begin{aligned}F(\xi, 0) &= 0, & G(\xi, 0) &= P_1(\xi) \\ \theta(\xi, 0) &= P_2(\xi) - S\xi & \text{for } \xi_i \leq \xi \leq \xi_j \\ G(\xi, 0) &= 0, & \theta(\xi, 0) &= 1 - S\xi & \text{for } \xi < \xi_i, \xi > \xi_j \\ F(\xi, \infty) &= \theta(\xi, \infty) = 0\end{aligned}\quad (9)$$

where

$$\begin{aligned}P_1(\xi) &= \varepsilon_2 \sin[A\pi^2(\xi - \xi_i)(\xi_j - \xi)] \\ P_2(\xi) &= 1 + \varepsilon_1 \sin[A\pi^2(\xi - \xi_i)(\xi_j - \xi)]\end{aligned}\quad (10)$$

Here ξ and η are the dimensionless distances along and perpendicular to the plate, respectively; F and G are the dimensionless velocities along ξ and η directions, respectively; Gr is the Grashof number with respect to L ; θ is the dimensionless temperature; M is the magnetic parameter; S is the ambient thermal stratification parameter; Pr is the Prandtl number; Ec is the dimensionless dissipation parameter; ψ is the Darcy number; Γ is the dimensionless inertial parameter; μ is the coefficient of viscosity; and A , ε_1 and ε_2 are constants. $\varepsilon_1 >$ or $<$ 0 according to whether the wall is being heated or cooled in the slot and $\varepsilon_2 >$

or $<$ 0 according to whether it is injection or suction in that region. Also, $\varepsilon_1 = 0$ when the entire wall is at a constant temperature T_1 . Similarly, $\varepsilon_2 = 0$ when there is no suction or injection at the wall.

It may be remarked that Eqs. (6)–(9) for $\varepsilon_1 = \varepsilon_2 = 0$ reduce to those of Chamkha [25]. For $M = \varepsilon = \varepsilon_1 = \varepsilon_2 = Ec = \psi^{-1} = \Gamma = \Omega = 0$, Eqs. (6)–(9) reduce to those of Eichhorn [2], Chen and Eichhorn [6], and Venkatachala and Nath [8]. Further for $M = \varepsilon_1 = \varepsilon_2 = Ec = \Omega = 0$, Eqs. (6)–(9) reduce to those of Chen and Lin [24].

The quantities of physical interest are the local skin friction and heat transfer coefficients and these can be expressed as;

$$\begin{aligned}C_{fx} &= \mu(\partial u / \partial y)_{y=0} / \rho(\nu Gr^{1/2} L^{-1})^2 \\ &= Gr^{-1/4} \partial F(\xi, 0) / \partial \eta \\ Nu_x &= -L(\partial T / \partial y)_{y=0} / (T_w - T_\infty) \\ &= -Gr^{1/4} \xi [\partial \theta(\xi, 0) / \partial \eta] (P_2(\xi) - S\xi)^{-1} \\ &\text{when } \xi_i \leq \xi \leq \xi_j \\ Nu_x &= -Gr^{1/4} \xi [\partial \theta(\xi, 0) / \partial \eta] (1 - S\xi)^{-1} \\ &\text{when } \xi < \xi_i, \xi > \xi_j\end{aligned}\quad (11)$$

where C_{fx} is the local skin friction coefficient and Nu_x is the local Nusselt number.

3. Method of solution

The partial differential equations (6)–(8) under the boundary conditions (9) have been solved by using an implicit iterative tridiagonal finite difference scheme similar to that of Blottner [29]. All the first order derivatives with respect to ξ are replaced by two-point backward difference formulae of the form;

$$\partial R / \partial \xi = (R_{i,j} - R_{i-1,j}) / \Delta \xi \quad (12)$$

where R is any dependent variable, rG or θ , and i and j are the node locations along the ξ and η directions, respectively. The second order partial differential equations (7) and (8) are discretized by using the three-point central difference formulae while the first-order differential equations are discretized by employing the trapezoidal rule. At each line of constant ξ , a system of algebraic equations are solved iteratively by using the well known Thomas algorithm (see Blottner [29]). The same process is repeated for the next ξ value and the equations are solved line by line until the desired ξ value is reached. A convergence criterion based on the relative difference between the current and the previous iterations is used. When this difference reaches 10^{-5} , the solution is assumed to have converged and the iterative process is terminated.

We have examined the effects of grid sizes $\Delta \eta$ and $\Delta \xi$, and the edge of the boundary layer η_∞ , on the solution. The results presented here are independent of grid size and η_∞ , at least up to the 4th decimal place.

4. Results and discussion

Eqs. (6)–(8) under the boundary conditions (9) have been solved numerically by using an implicit finite-difference scheme as described earlier. We have compared the value of the heat transfer parameter $-\theta'(\xi, 0)$ for $\varepsilon = \psi^{-1} = \Gamma = 0$ (constant density medium), $\Omega = 0$ (vertical plate), $M = 0$ (without magnetic field), $\varepsilon_1 = 0$ (isothermal surface and $\varepsilon_2 = 0$ (without mass transfer) with those of Eichhorn [2], Chen and Eichhorn [6] and Venkatachala and Nath [8], who used the series solution method, the local non-similarity method and the finite-difference method, respectively, in their analyses. For large $Pr (= 6)$, the local non-similarity results of Chen and Eichhorn [6] are in very good agreement with the present results. However, for small values of Pr ($Pr = 0.7$) and for $\xi \geq 0.6$, the local non-similarity method slightly over-estimates the heat transfer. On the other hand, the series solution results of Eichhorn [2] are in good agreement with those of the finite-difference method for $\xi \leq 0.6$, but beyond this value they differ significantly. The series solution method for $\xi \geq 0.6$ under-estimates the heat transfer results.

It may be noted that for the computations, we have taken two slots located in the intervals [0.1, 0.3] and [0.5, 0.7] and the constant (A) in the wall temperature and mass transfer distributions to be equal to 1. The suction/injection is applied only in these two intervals. Also the wall is heated or cooled in these intervals only.

Figs. 2 and 3 present the effects of non-uniform wall temperature in two slots ($\varepsilon_1 \neq 0$) and the thermal stratification parameter S on the local skin friction coefficient ($Gr^{1/4}C_{fx}$) and the local Nusselt number ($Gr^{-1/4}Nu_x$) in the absence of suction / injection ($\varepsilon_2 = 0$) when $Ec = M = 1$, $Pr = 5.4$,

$\varepsilon = 0.4$, $\Gamma = 10$, $\psi = 1$, $\Omega = \pi/3$. The corresponding results for the constant temperature over the entire surface ($\varepsilon_1 = 0$) are shown in Figs. 4 and 5. It is evident from these figures that the non-uniform wall temperature induces a complicated behaviour in the variation of the skin friction and the heat transfer with the streamwise distance ξ . Since the change in the wall temperature takes place in two slots only, the effect is most pronounced within and in the vicinity of these slots (especially in and near the second slot). For slot heating ($\varepsilon_1 > 0$), the skin friction coefficient ($Gr^{1/4}C_{fx}$) is higher than that of the slot cooling ($\varepsilon_1 < 0$) for all values of the stratification parameter S . On the other hand, the skin friction decreases with increasing S for both heating and cooling of the wall in the slot. The reason for the above trend is that for a fixed S the heating of a section of the surface ($\varepsilon_1 > 0$) makes the liquid near the surface less viscous, which offers less resistance to the liquid motion and the fluid is accelerated. Consequently, the skin friction coefficient increases. The effect of slot cooling ($\varepsilon_1 < 0$) is opposite to that of slot heating ($\varepsilon_1 > 0$). Also the effect of the stratification parameter S becomes more pronounced with increasing streamwise distance ξ , because the wall temperature decreases with ξ . Hence the temperature difference between the wall and fluid near the wall increases. The local Nusselt number ($Gr^{-1/4}Nu_x$) shows a more complicated trend in the vicinity of the second slot. The effect of cooling on the heat transfer is more pronounced than that of heating, because due to the cooling of the wall, the temperature difference near the surface increases, which causes considerable changes in the heat transfer near the second slot. Also the direction of the heat transfer changes in and near the second slot. Since the wall temperature T_1 is higher than that of the fluid near the wall, the heat is transferred from the wall

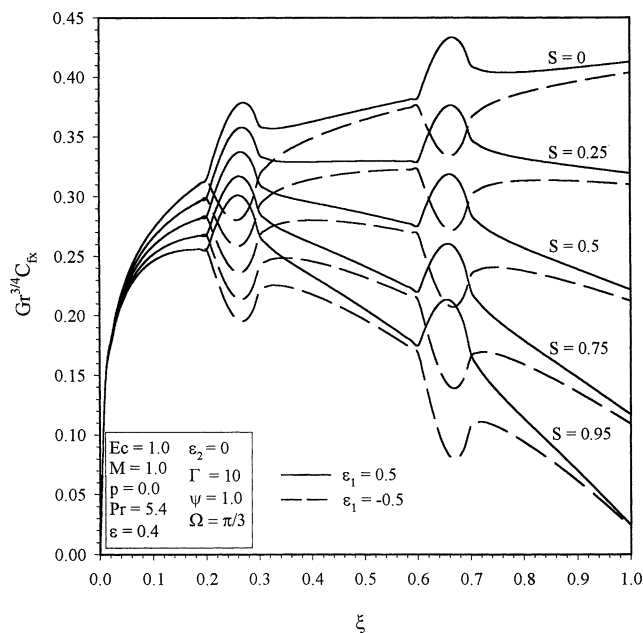


Fig. 2. Skin friction coefficient, $Gr^{1/4}C_{fx}$, for non-uniform wall temperature in the slots for several values of the thermal stratification parameters, S .

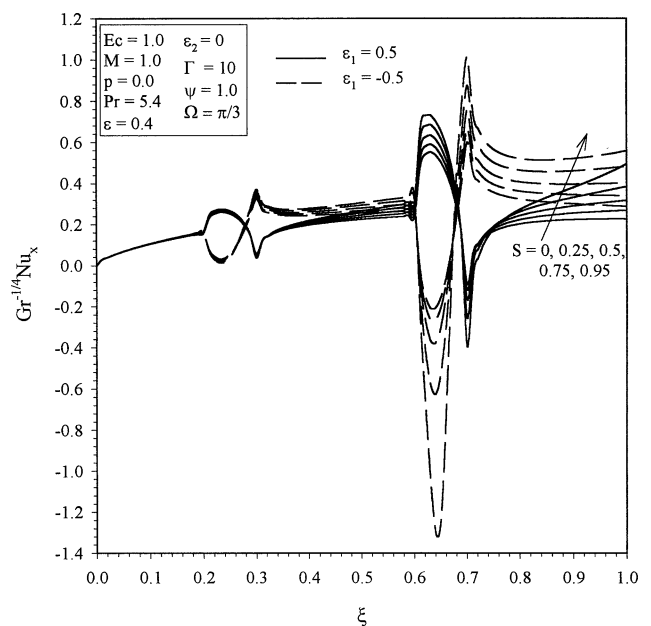


Fig. 3. Nusselt number, $Gr^{-1/4}Nu_x$, for non-uniform wall temperature in the slots for several values of the thermal stratification parameter, S .

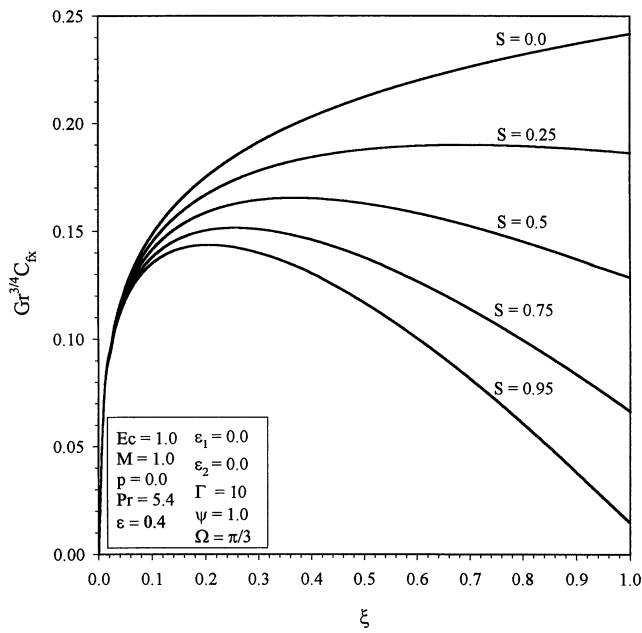


Fig. 4. Skin friction coefficient, $Gr^{1/4}C_{fx}$, for uniform wall temperature on the entire surface for several values of the thermal stratification parameter, S .

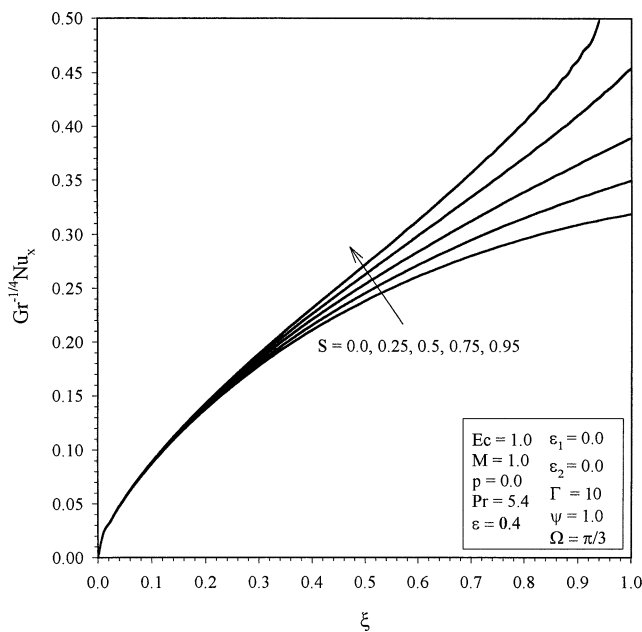


Fig. 5. Nusselt number, $Gr^{-1/4}Nu_x$, for uniform wall temperature on the entire surface for several values of the thermal stratification parameter, S .

to the fluid. When the wall is cooled, the temperature of the surrounding fluid becomes more than that of the wall. Hence the heat is transferred from the fluid to the wall. In the regions $0.3 \leq \xi \leq 0.6$ and $0.7 < \xi \leq 1.0$, the Nusselt number for wall cooling ($\epsilon_1 < 0$) is more than that of the wall heating ($\epsilon_1 > 0$), because temperature difference near the wall increases with cooling. Hence the heat transfer increases due to wall cooling.

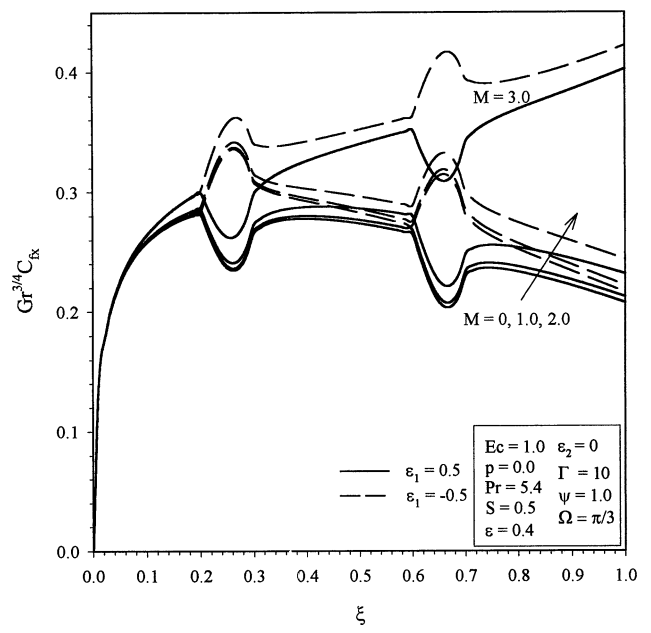


Fig. 6. Skin friction coefficient, $Gr^{1/4}C_{fx}$, for non-uniform wall temperature in the slots for several values of the magnetic parameter, M .

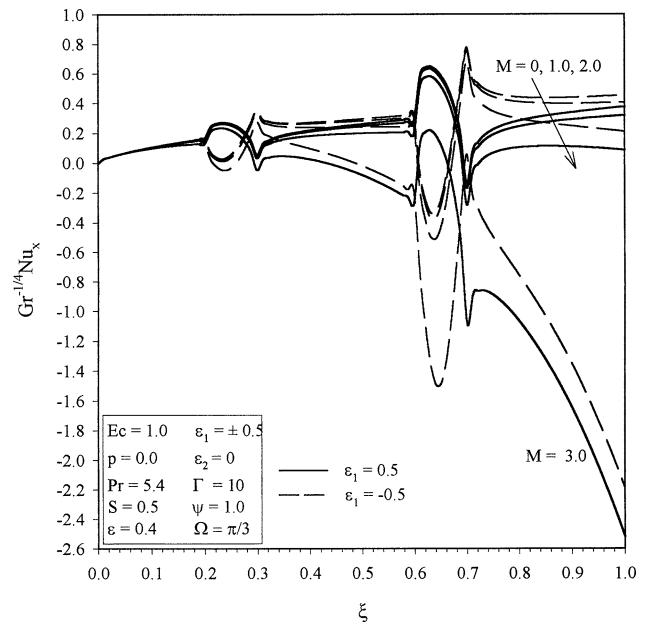


Fig. 7. Nusselt number, $Gr^{-1/4}Nu_x$, for non-uniform wall temperature in the slots for several values of the magnetic parameter, M .

In Figs. 6 and 7, the effects of wall heating/cooling in the slots without mass transfer ($\epsilon_2 = 0$) on the skin friction coefficient ($Gr^{1/4}C_{fx}$) and the Nusselt number ($Gr^{-1/4}Nu_x$) for various values of the magnetic parameter M , when $Ec = 1$, $Pr = 5.4$, $S = 0.5$, $\epsilon = 0.4$, $\Gamma = 10$, $\psi = 1$, $\Omega = \pi/3$ are shown. Since the effect is qualitatively similar to that of the previous figures, it is not discussed here.

The dimensionless velocity components in the x and y directions, $F(\xi, \eta)$ and $G(\xi, \eta)$, and the dimensionless temperature $\theta(\xi, \eta)$, for the heating/cooling of the wall

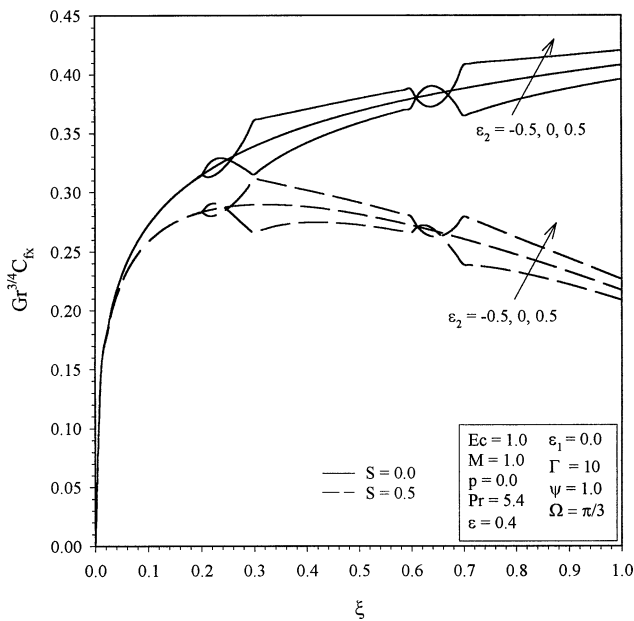


Fig. 8. Skin friction coefficient, $Gr^{1/4}C_{fx}$, for non-uniform mass transfer in the slots.

temperature in the slots for several values of the stratification parameter S when $Ec = M = 1$, $Pr = 5.4$, $\varepsilon = -0.4$, $\varepsilon_2 = 0$, $\Gamma = 10$, $\psi = 1$, $\Omega = \pi/3$, $\xi = 0.25$ have been calculated. It is found that in general, the velocities and temperature (F, G, θ) for the wall cooling ($\varepsilon_1 < 0$) are less than those of wall heating ($\varepsilon_1 > 0$). Since the wall cooling has a stabilizing effect, the velocities and the temperature are reduced by wall cooling. Since the increase in the thermal stratification parameter S reduces the wall temperature and hence the thermal buoyancy forces, the velocities and the temperature are reduced due to an increase in the stratification parameter.

The effect of multiple slot suction/injection ($\varepsilon_2 \neq 0$) on the local skin friction coefficient ($Gr^{1/4}C_{fx}$) and the local Nusselt number ($Gr^{-1/4}Nu_x$) for $Ec = M = 1$, $Pr = 5.4$, $\varepsilon = 0.4$, $\varepsilon_1 = 0$, $\Gamma = 10$, $\psi = 1.0$, $\Omega = \pi/3$ is displayed in Figs. 8 and 9. Since the effect of suction ($\varepsilon_2 < 0$) is to suck away the warm fluid near the plate; thus it decreases both the momentum and thermal boundary layer thicknesses, the skin friction and heat transfer coefficients increase due to suction. On the other hand, injection ($\varepsilon_2 > 0$) increases the momentum and thermal boundary layer thicknesses. Hence both the skin friction and the heat transfer are reduced by injection, but it is not a mirror reflection of suction.

5. Conclusions

The results indicate that the non-uniform wall temperature and surface mass transfer in certain sections of the vertical surface exert a strong influence on the local heat transfer and skin friction. When the wall temperature is reduced in the slots, there is a change in the direction of the heat transfer. The skin friction and the heat transfer increase due

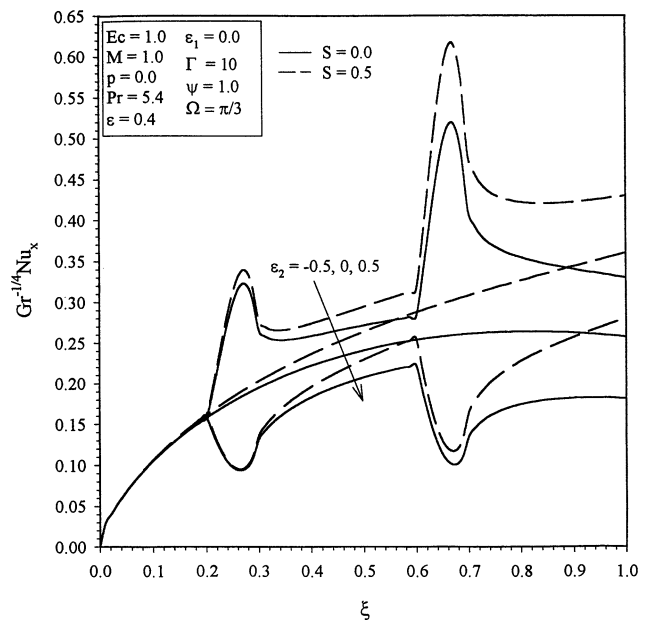


Fig. 9. Nusselt number, $Gr^{-1/4}Nu_x$, for non-uniform mass transfer in the slots.

to suction, but reduce due to injection. The heat transfer changes very significantly in the region of the second slot. The effects of surface suction and injection or the surface heating and cooling in the slots, are not a mirror reflection of each other. The thermal stratification and magnetic parameters produce significant changes on the skin friction and the heat transfer.

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